Group A: Use inclusion-exclusion to provide a combinatorial proof to the following.

$$\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n = \begin{cases} n! & \text{if } k=n; \\ 0 & \text{if } k>n. \end{cases}$$

Hint: Let $S$ be a set of $m$ elements. Then $m^n$ is the number of ways to assign numbers $1,\ldots,n$ to the elements in $S$. In other words, $m^n$ is the number of mappings from $\{1,\ldots,n\}$ to $S$.

Group B: Use inclusion-exclusion to provide a combinatorial proof to the following.

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}\binom{m+n-i}{k-i} = \begin{cases} \binom{m}{k} & \text{if } m\geq k; \\ 0 & \text{if } m<k. \end{cases}$$

Group C: Let Euler's $\phi$ function $\phi(n)$ be defined as the number of integers from 1 to $n$ that are relatively prime to $n$. If $n$ is an integer whose unique prime factorization is

$$n = p_1^{e_1}p_2^{e_2}\cdots p_r^{e_r},$$

then use inclusion-exclusion to prove that

$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_r}\right).$$