# Midterm 1 Practice Problems

*These are a collection of practice problems for the first midterm exam. Some of the problems are more computationally involved than those on the exam will be. The actual exam will of course be much shorter than this. If you can do these problems (without looking at solutions), there is a high probability that you will do well on the exam.*

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**1.** How many arrangements are there of the word PROBABILITY?

**2.** Suppose that who events $A$ and $B$ satisfy $P(A) = 0.4$, $P(B) = 0.3$ and $P(\overline{(A \cup B)}) = 0.42$. Are $A$ and $B$ independent?

**3.** There are six men and seven women in a ballroom dancing class. If four men and four women are chosen and paired off, how many pairings are possible?

**4.** There are 8 cards in a hat:

$$\{1\heartsuit, 1\spadesuit, 1\diamondsuit, 1\clubsuit, 2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit\}$$

You draw one card at random. If its rank is 1 you draw one more card; if its rank is two you draw two more cards. Let X be the sum of the ranks on the 2 or 3 cards drawn. Find E(X).

**5.** Would you rather take a multiple-choice test or a full recall test? If you have no knowledge of the test material, you will score zero on a full recall test. However, if you are given 5 choices for each multiple choice question, you have at least one chance in five of guessing each correct answer! Suppose that a multiple choice exam contains 100 questions,each with 5 possible answers, and guess the answer to each of the questions.

(a) What is the expected value of the number $Y$ of questions that will be answered correctly?

(b) Find the standard deviation of $Y$.

(c) Calculate the intervals $\mu \pm 2\sigma$ and $\mu \pm 3\sigma$.

(d) If the results of the exam are curved so that 50 correct answers is a passing score, are you likely to receive a passing score? (What does Chebyshevs inequality give you?)

**6.** For a bill to make it to the office of the president, it must be passed by both the House and the Senate. Suppose that of all bills, 60 percent pass the House, 80 percent pass the Senate, and 90 percent pass at least one of the two. What is the probability that the next bill will make it to the office of the president?

**7.** A student attends a career fair and visits booths of different companies. Their chance of getting an interview with a company after they visit a booth depends on whether they took APMA 1650 or not. If the student took 1650 then the chance of being offered an interview is $p = 95\%$. If the student didn't take 1650 then they have $p = 15\%$ chance of being offered an interview.

(a) Let $X$ record the number of booths a student visits until they receive an interview. Give the probability distribution function for $X$ in terms of $p$.

(b) On average, how many booths must student who took 1650 visit before getting an interview? How about a student who hasn't?

(c) Suppose each student visits 5 booths during a career fair, what is the probability that a student who took 1650 will not get an interview? What is the probability that a student who didn't take 1650 will get an invitation?

**8.** Suppose a pond contains two colors of fish, 6 red and 9 blue. You catch one fish in a net equally likely at random from all fish in the pond, note its color and then return the fish along with 2 more of the same color into the pond and then repeat.

(a) What is the probability that the first fish caught is Red and thenext two are Blue?

(b) What is the probability that there are more Red fish than Blue fish after 3 iterations?

**9.** For two events $A$ and $B$, show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

**10.** Suppose that $X \sim \text{Binomial}(n, 0.5)$. Find the distribution function of $Y = 2X$.

**11.** Let A and B be two events. Suppose the probability that neither A or B occurs is 2/3. What is the probability that one or both occur?

**12.** Let $C$ and $D$ be two events with $P(C) = 0.25$, $P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(\bar{C} \cap D)$?

**13.** Suppose 100 people all toss a hat into a box and then proceed to randomly pick out a hat. What is the expected number of people to get their own hat back?

**14.** A pair of two cards are drawn successively from a deck of 52 cards. Find the probability that the second card is higher in rank than the first card. Hint: Show that $1 = P(\text{higher}) + P(\text{lower}) + P(\text{same})$ and use the fact that $P(\text{higher}) = P(\text{lower})$.

**15.** A started motor used in a space vehicle has a high rate of reliability and was reputed to start on any given occasion with probability .99999. What is the probability of at least one failure in the next 10,000 starts?

**15.** Articles coming through an inspection line are inspected by two successive inspectors. When a defective article comes through the inspection line, the probability that it gets by the first inspector is .1. The second inspector will miss five out of ten of the defective items that get pastthe first inspector. What is the probability that a defective item gets by both inspectors?

**16.** Suppose a class has 150 students.

(a) A mad professor decides to pick 50 students equally likely at random and give those 50 students a free A (I would never do this). Suppose you are in the class, what is the probability that you will be one of the 50 students chosen and hence receive a free A?

(b) Exams are handed back one by one in a random order (i.e. an ordering of the 150 students is chosen equally likely at random from all possible orderings of students and the exams are handed back in that order). If Alice and Bob are two students in class, what is the probability that Alice will get her test back before Bob?

**17.** I have a bag with 3 coins in it. One of them is a fair coin, but the others are biased trick coins. When flipped, the three coins come up heads with probability 0.5, 0.6, 0.1 respectively.
Suppose that I pick one of these three coins uniformly at random and flip it three times.

(a) What is P(HTT)? (That is, it comes up heads on the first flip and tails on the second and third flips.)

(b) Assumimg that the three flips, in order, are HTT, what is the probability that the coin that I picked was the fair coin?

**18.** Ten teams are playing in a tournament. In the first round, the teams are randomly assigned to games 1,2,3,4,5. In how many ways can the teams be assigned to the games? How about $2n$ teams and $n$ games?

**19.** If $Y \sim$ Binomial$(n, p)$, show that

$$P(Y > 1 | Y \geq 1) = \frac{1 - (1-p)^n - np(1-p)^{n-1}}{1 - (1-p)^n}$$