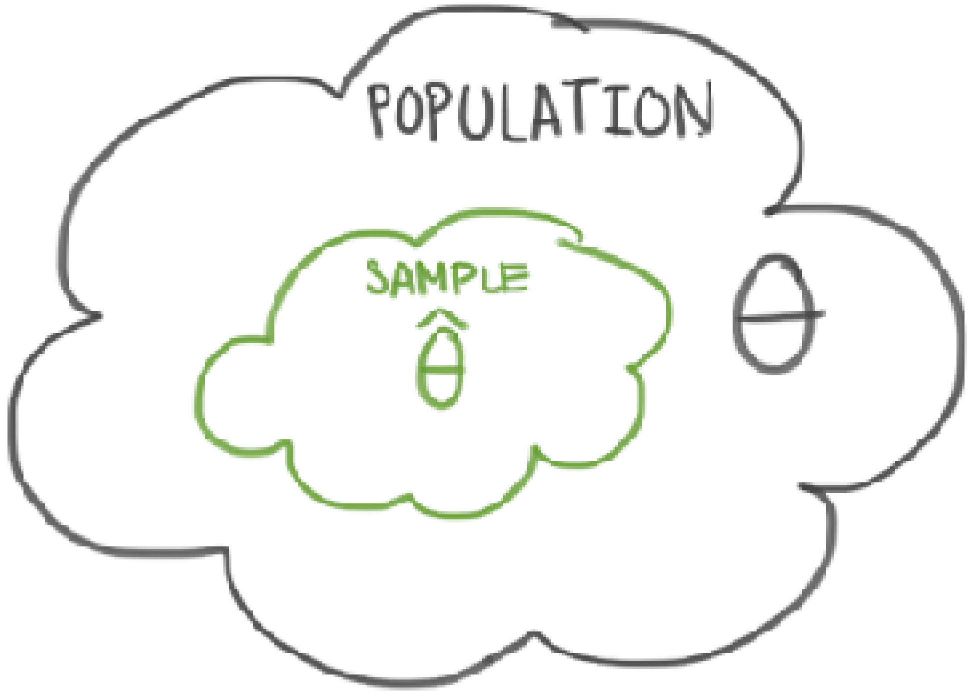
The Final Stretch

janet_qi@brown.edu

The New Stuff

ESTIMATION

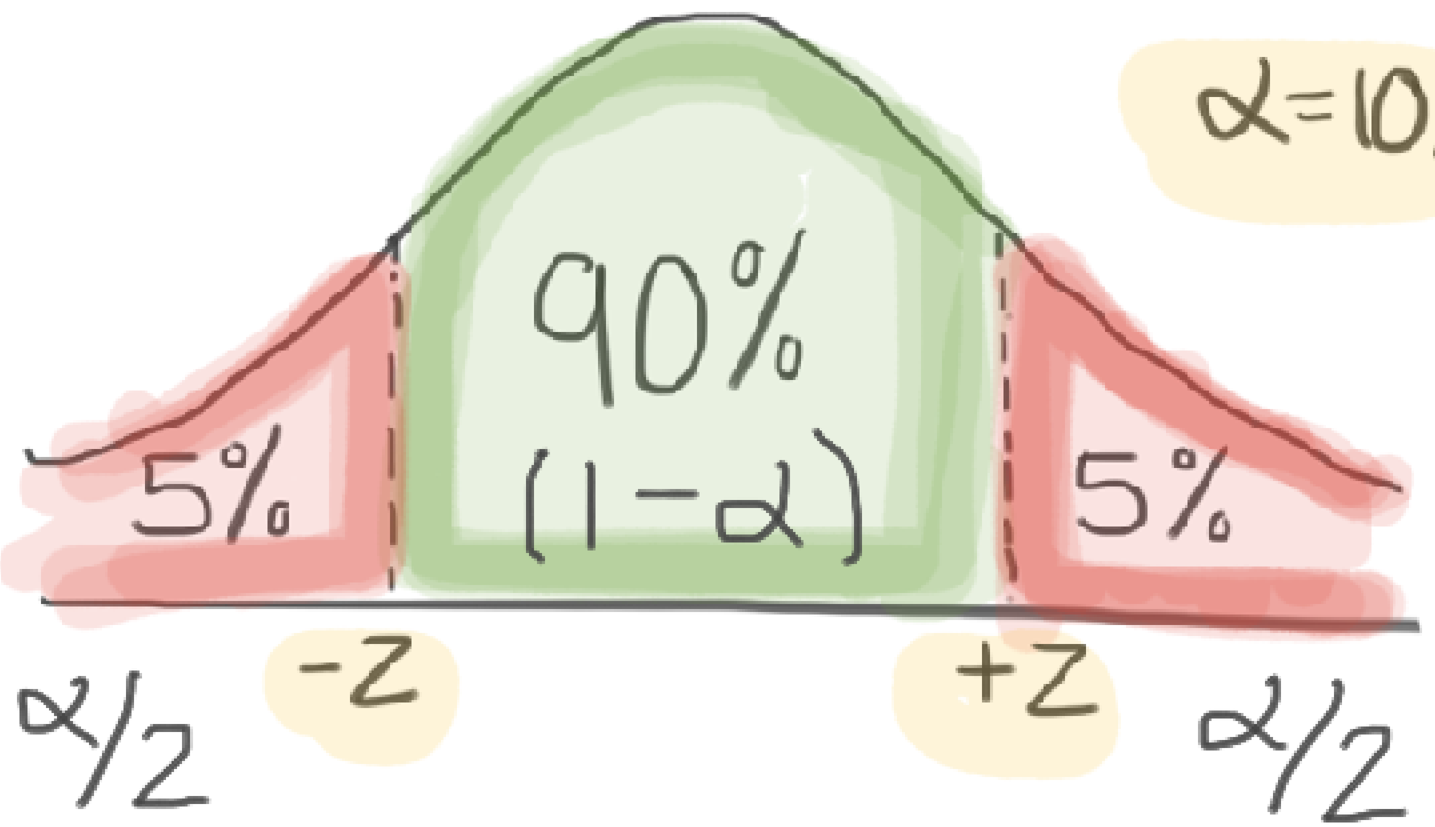


$\Rightarrow$ depending on how we calculate $\hat{\theta}$, how accurate can we get to θ???

$\Rightarrow$ **BIAS** $\quad \text{bias}(\hat{\theta}) = E[\hat{\theta}] - \theta$

$\Rightarrow$ **MEAN SQUARE ERROR** $\quad MSE(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right] = \text{var}[\hat{\theta}] + \left(\text{bias}(\hat{\theta})\right)^2$

Confidence Intervals



$\alpha = 10\%$

$\Rightarrow \dfrac{\hat{\mu} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$\Rightarrow P\left(-Z < \dfrac{\hat{\mu} - \mu}{\sigma/\sqrt{n}} < Z\right) = 90\%$ rearrange

$= P\left(-Z\left(\frac{\sigma}{\sqrt{n}}\right) - \hat{\mu} < -\mu < Z\left(\frac{\sigma}{\sqrt{n}}\right) - \hat{\mu}\right) = 90\%$ $\times -1$

$= P\left(Z\left(\frac{\sigma}{\sqrt{n}}\right) + \hat{\mu} > \mu > \hat{\mu} - Z\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 90\%$

$\Rightarrow$ if sample data: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

if $n \leq 30$

$\Rightarrow$ if n is "small"

$\Rightarrow$ use t-dist

↳ different value table from Zs

EFFICIENT

An unbiased estimator is said to be the **most efficient** if it has the **smallest variance** among all possible unbiased estimators

CONSISTENT

$$\lim_{n\to\infty} P(|\hat{\theta} - \theta| > \epsilon) = 0 \rightsquigarrow \lim_{n\to\infty} \hat{\theta} = \theta$$

↳ WLLN : $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \beta_i^{k} = E[\beta^{k}]$

↳ $\lim_{n\to\infty} \frac{*}{n} = 0$

SUFFICIENT

$P(X_1 = x_1 \dots X_n = x_n \mid \hat{\theta}(X_1 \dots X_n) = u)$ does not depend on θ

↳ $L(\theta) = L(X_1 \dots X_n \mid \theta) = g(u,\theta) \cdot h(X_1 \dots X_n)$

METHOD OF MOMENTS

$\Rightarrow$ relating sample and theoretical moments

THEORETICAL		SAMPLE
$E[X]$	$\longrightarrow$	$\frac{1}{n}\sum_{i=1}^{n} X_i$
$E[X^2]$	$\longleftrightarrow$	$\frac{1}{n}\sum_{i=1}^{n} X_i^2$
$\vdots$		
$E[X^K]$	$\longleftrightarrow$	$\frac{1}{n}\sum_{i=1}^{n} X_i^K$

often in terms of parameters

1. figure out $E[X]$, $E[X^2]$, etc.
   (however many equations you need)
   ↳ use what you know about distribution
   - ↳ variance
   - ↳ integrate given PDF
2. Solve system of equations for parameters you're looking for

Maximum Likelihood Estimator

⇒ take the derivative of some expression

→ not to be confused w/ MSE

① usually given $X_1 \ldots X_n$ where $f(x|\star) =$

② $L(\star) = P(X_1 = x_1, \ldots X_n = x_n | \star)$ ← "joint probability in terms of $\star$"

③ $L(\star) \Rightarrow \prod_{i=1}^{n} f(x_i|\star)$ because X_is are IID

④ note: $\ln(AB) = \ln(A) + \ln(B)$

⑤ take derivative $\frac{d}{d\star}$ ⇒ set = 0 ⇒ solve for $\star$

Invariance Property

$\Rightarrow$ if $t(\theta)$ is a one-to-one function of θ and if $\hat{\theta}$ is the MLE for θ, then the MLE of $t(\theta)$ is given by

$\widehat{t(\theta)} = t(\hat{\theta})$ ↳ plug in $\hat{\theta}$ into t function

Other Useful Things

$\Rightarrow \ln(AB) = \ln(A) + \ln(B)$

$\Rightarrow$ expected values and variances of common distributions

$\Rightarrow$ t-dist = normal + small n

$\Rightarrow E[f(x)] = \int f(x) \cdot pdf(x)\,dx$ OR $\sum f(x) \cdot pdf(x)$

Ex: let $\mathbb{X} = (X_1 \dots X_n)$ be a random sample of size n w/ mean μ and variance σ^2

$\Rightarrow$ let $\hat{s}^2(\mathbb{X}) = \frac{1}{n} \sum_{i=1}^{n} \left(X_i - \frac{1}{n} \sum_{i=1}^{n} X_i\right)^2$ **AND** suppose $E[X_i^4] < \infty$

$\Rightarrow$ show $\hat{s}^2$ is consistent

Ex: let $X \sim \text{Gamma}(\alpha, \beta) \Rightarrow E[X] = \alpha\beta$ and $Var[X] = \alpha\beta^2$

$\Rightarrow$ find MME for α and β

Ex: let $Y \sim \text{geometric}(\theta)$ where $f(y|\theta) = \theta(1-\theta)^{y-1}$ for $y = 1, 2, 3 \ldots$

$\Rightarrow$ find MLE for θ