THE FINAL STRETCH

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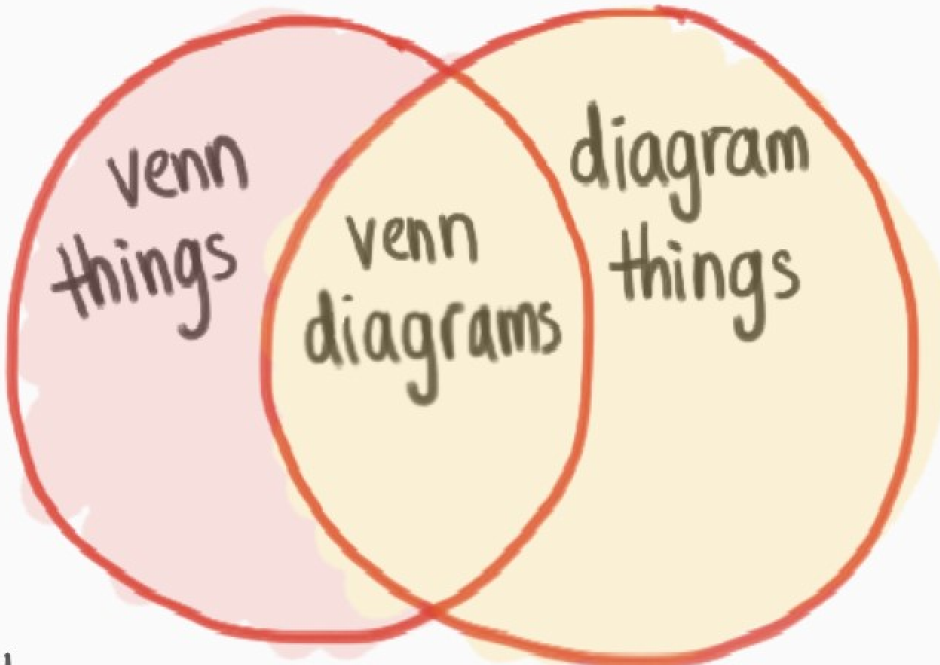
THE OLD STUFF

SET THEORY

- ⇒ $A \cup B$ = A or B or both = Union
- ⇒ $A \cap B$ = A and B = intersect
- ⇒ $\overline{A}$ = complement of A = not A
- ⇒ $A \cap B = \varnothing$ → mutually exclusive events
- ⇒ De Morgans ⇒ $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
  ⇒ $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
- ⇒ distributive laws ⇒ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  ⇒ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

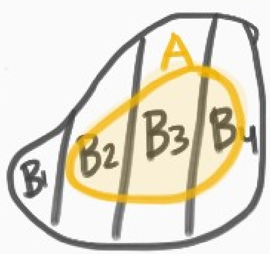


Fundamentals of Probability

⇒ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

↳ if you have $P(A \cup B \cup C \cup \ldots)$ don't forget to "add back" parts that have been double subtracted

⇒ $P(A) = 1 - P(\bar{A})$

⇒ 

⇒ $A = (A \cap B_1) \cup (A \cap B_2) \cup \ldots$

Discrete Prob. + Combinatorics

$\Rightarrow \dfrac{\text{how many I want}}{\text{how many total}}$

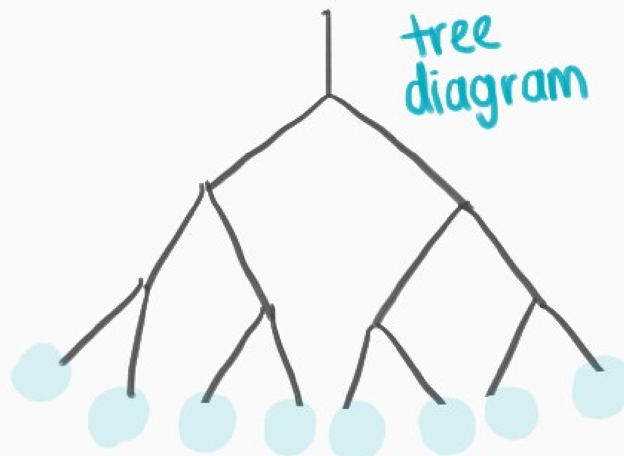
$\Rightarrow n! = (n)(n-1)(n-2)\dots 1$

$\Rightarrow$ permutation $= P_r^n = \dfrac{n!}{(n-r)!}$ = order MATTERS

$\Rightarrow$ partition $= \dfrac{n!}{n_1!\, n_2! \dots n_k!}$ = n things into k groups w/ $n_1 \dots n_k$ each

$\Rightarrow$ combination $= \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$ = order DOESN'T matter

Conditional Probability



$\Rightarrow P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

↳ rearrange this to solve for what you need

↳ sometimes need to find A using law of total probability

↳ $P(A) = \sum_{i=1}^{k} P(A|B_i) \cdot P(B_i)$

$\Rightarrow$ Bayes: $P(B|A) = \dfrac{P(A|B) \cdot P(B)}{P(A)}$

$\Rightarrow$ independent $\Leftrightarrow P(B|A) = P(B)$

$\Leftrightarrow P(A \cap B) = P(A) \cdot P(B)$

DISCRETE RV

$\Rightarrow$ see formula sheet

↳ know what parameters represent

$\Rightarrow E[X+Y] = E[X] + E[Y]$

$E[cX] = c \cdot E[X]$

$\Rightarrow Var[cX] = c^2 Var[X]$

$Var[X+c] = Var[X]$

$Var[X] = E[X^2] - (E[X])^2 = E[(X - E[X])^2]$

WLLN

: let $X_1 \ldots X_n$ be iid, $E[X_i] = \mu$, $Var[X_i] = \sigma^2$

↳ $P(|\bar{X} - \mu| > \epsilon) < \frac{\sigma^2}{n \cdot \epsilon^2}$

CHEBYSHEV

: $P(|\underset{\uparrow \text{RV}}{X} - \mu| \geq k\sigma) \leq \frac{1}{k^2}$

Useful Formulas

- $\text{CDF} = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$ PDF
- $\int_A f(x)\,dx = \underline{1}$ <u>OR</u> $\int\int_A f(x,y)\,dx\,dy = 1$
- $E[X^2] = \int x^2 \cdot f(x)\,dx$ (single var) <u>OR</u> $\int x^2 \cdot f_x(x)\,dx$ (multivar) <u>OR</u> $\int\int x^2 \cdot f(x,y)\,dy\,dx$ (multivar)
- $f_x(x) = \int f(x,y)\,dy$ (bounds in terms of x)
- $f(x|y) = \dfrac{f(x,y)}{f_y(y)}$

Useful Formulas

- $E[X|Y] = \int x \cdot f(x|y)\,dx$
- $E[X] = E\big[E[X|Y]\big]$
- X ind Y IFF $f_X(x) \cdot f_Y(y) = f(x,y)$
- $Cov(X,Y) = E\big[(X-\mu_X)(Y-\mu_Y)\big] = E[XY] - E[X]E[Y]$
- X ind Y $\rightarrow Cov(X,Y)=0$ BUT $Cov(X,Y)=0 \not\Rightarrow$ X ind Y
- $\rho_{X,Y} = \dfrac{cov(X,Y)}{\sigma_X \sigma_Y} \Rightarrow -1 \le \rho_{XY} \le 1$

HW5, #5

- CLT $\Rightarrow Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$ as $n \to \infty$
  - $X_i = \begin{cases} 1 & \text{if } \checkmark \\ 0 & \text{if } X \end{cases} \Rightarrow$ μ = proportion = p
    $var(X_i) = pq \Rightarrow \sigma = \sqrt{pq}$

Multivariate Stuff

$$
\begin{tikzcd}
 & & & f(y|x) = \frac{f(x,y)}{f_x(x)} \\
 & & f_x(x) \arrow[ur] \arrow[dr, dashed, "\text{if indep.}"] & \\
F(x,y) \arrow[r, "dxdy"] & f(x,y) \arrow[ur, "\int dy"] \arrow[dr, "\int dx"'] & & f_x(x) \cdot f_y(y) = f(x,y) \\
 & & f_y(y) \arrow[ur, dashed, "\text{if indep.}"'] \arrow[dr] & \\
 & & & f(x|y) = \frac{f(x,y)}{f_y(y)}
\end{tikzcd}
$$

Ex: $f(x,y) = \begin{cases} 1/2 & \text{if } 0 \le x \le y \le 2 \\ 0 & \text{if otherwise} \end{cases} \Rightarrow f(x|y)? \Rightarrow E[x]?$

Ex: 5 unlabeled bins. Bin 1 w/ P(broken) = 0.05

Bins 2-5 w/ P(broken) = 0.02

⇒ You pick 3 things from a random bin and 1 is broken

⇒ P(you chose bin 1)?

Ex: 10 hunters fire at the same time at a flock of 10 ducks. Each hunter picks one duck and has an accuracy of 10% (they're not great).

$\Rightarrow$ E[# of ducks that escape unhurt]?

Ex: $f(x,y) = \begin{cases} cx^2y & \text{if } -1 \le x \le 1,\ 0 \le y \le \min\left\{\sqrt{|1-x|}, \sqrt{|1+x|}\right\} \\ 0 & \text{if otherwise} \end{cases}$

$\Rightarrow$ find c that makes $f(x,y)$ a valid pdf