APMA 1650 Final Exam Checklist

The exam will be broken down into 50% on new material (estimators and confidence intervals) 25% on material from midterm 1 and 25% material from midterm 2. Some fundamental concepts like random variables, expectation, conditional probabiltiy, law of large numbers and central limit theorem, etc are cummulative and permeate everything we have done in the class.

Stuff from Midterm 1 (25% of the exam)

1. Basics of set theory: Make sure you understand how to manipulate sets and the various operations that can be performed.
   (a) Union, intersection, complement, mutually exclusive, Venn diagrams
   (b) DeMorgans Law and algebraic properties of sets
2. Fundamentals of probability
   (a) laws of addition, complement, difference
   (b) how to calculate probabilities by splitting up events
3. Discrete probability and combinatorics
   (a) Sample point method, probability tables
   (b) Equally likely events, counting, rule of products, permutations, combinations, partitions
   (c) Make sure you understand the formulas for permutations, combinations and partitions
4. Conditional probability
   (a) Know the definition and what it means intuitively
   (b) Law of multiplication, law of total probability, probability trees
   (c) Independence, what does it mean, how to check it
   (d) Bayes rule, and inverting conditional probabilities, Bayesian inference
5. Discrete random variables and distributions
   (a) Distribution function
   (b) Expected value and variance know their properties and how use use them
   (c) Special distributions: Bernoulli, binomial, geometric (both versions), Poisson. Know their properties and what they describe.
6. Weak law of large numbers and Chebyshev
   (a) What is the weak law of large numbers and what does it say?
   (b) Chebyshev's inequality, what does it mean and how to apply it to bound probabilities.
   (c) How to prove the weak law of large numbers from Chebyshev's inequality

Stuff from Midterm 2 (25% of the exam)

1. Continuous random variables and distributions
   (a) Probability density function, cumulative distribution functions
   (b) Expected value and variance again
   (c) Special Distributions: Uniform distribution, exponential distribution (how it relates to geometric), normal distribution.
   (d) Distribution method. What is the distribution of $h(X)$?
2. Central limit theorem
   (a) Applications to sampling and repeated measurements, voting and surveying populations
   (b) compare to weak law of large numbers
3. Multivariate distributions
   (a) Joint densities and cdfs, interated integrals and integrating over regions
   (b) Marginals, conditional densities, independent random variables
   (c) Covariance and correlation. Relation to independence. Can a two random variables have no correlation, but be dependent? Do you have an example?
   (d) Conditional expectation and conditional variance. How to use them to compute unconditional variance and expectation (law of total expectation)

New material (50% of the exam)

1. Sampling and estimation
   (a) Point estimators, what are they? What are some examples?
   (b) Bias of an estimator. How to make a biased estimator unbiased.
   (c) Mean square error, it's relation to Variance and Bias of an estimator, know how to compute
2. Analysis of estimators and Confidence intervals for a population mean
   (a) What is a $(1 - \alpha)$ confidence interval of a population mean? How does is relate to a point estimator of a mean.
   (b) Pivotal quantities. What are they? How to use them to calculate confidence intervals.
   (c) How to use the CLT to get confidence intervals if the sample size is large.
   (d) How to use the t-distribution if the sample population is normal (and the sample size is not large)
   (e) You don't need to know how to estimate confidence intervals for population variances or for differences of population means.
3. Efficiency, Consistency, and sufficiency

(a) How to show that one estimator is better than another.

(b) What does it mean to show an estimator is consistent. How to use the MSE to show consistency.

(c) How to use the weak law of large numbers to show consistency.

(d) What is convergence in probability?

(e) Properties of convergence in probability and how to use it to show consistency.

(f) What is a sufficient statistic? How to use the factorizaton method to show a statistic is sufficient.

4. Methods of estimation

   (a) Method of moments, how to use to estimate multiple parameters (not just one)

   (b) Maximum likelihood estimators (MLE). How to compute them. How are they related to sufficient statistics?

   (c) Why taking the logarithm makes things easier (also very useful in numerics and semi-definite programing for the computer science folks)

   (d) Invariance property. How to use it to caclulate more complicated MLE's.