Announcements

- The calculus prep assignment has been modified.
  → new due date: Saturday 1/23 11:59 pm.

Set Theory Continued.

Set operations , $\cup$, $\cap$, $(\ )^c = \overline{(\ )}$

union intersection complement.

With these operations the subsets of S form a Boolean algebra.

Ex

$$[(A \cup B) \cap C] \cup (A \cup C \cup D)$$

associativity.

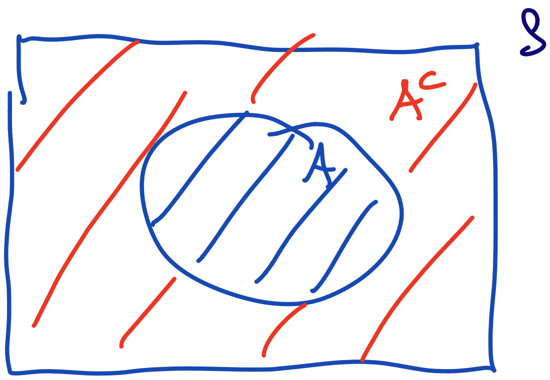
Distributive Law

$A, B, C \subset S$

① $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

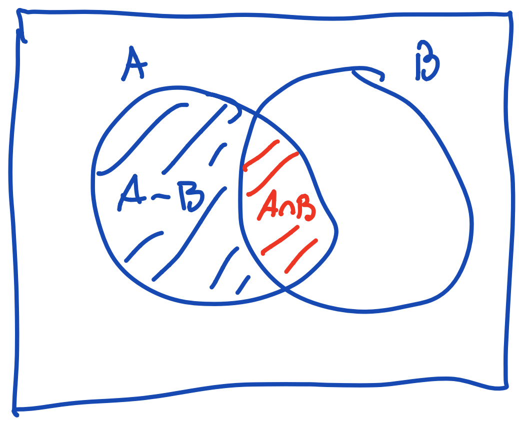
② $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Decompose events.



$S = A \cup A^c$

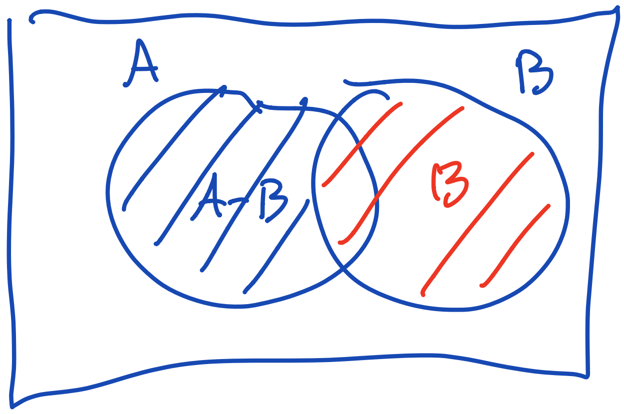
↑ disjoint



$A = (A - B) \cup (A \cap B)$

↑ disjoint.

$A - B = A \cap B^c$



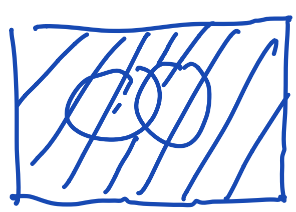
$A \cup B = (A - B) \cup B$

De Morgan's Law

① $(A \cup B)^c = A^c \cap B^c$

$\cup$ ↑ union; $\cap$ ↑ intersection

② $(A \cap B)^c = A^c \cup B^c$



① $\Longleftrightarrow$ ② Using $(A^c)^c = A$.

Countability and uncountability.

- Cardinality of a set A

$|A| =$ # of elements of the set.

- $|A| < \infty$ the set is finite. $\{1, 2, 3\}$
- $|A| = \infty$ the set is infinite. $, \mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{Q}$

  $\uparrow$ rational.

There are <u>two</u> types of infinity.

countable $\rightarrow$ the elements of the set can be turned into a sequence.

$A = \{a_1, a_2, a_3, \ldots\}$

uncountable $\rightarrow$ if it's not countable

<u>Ex</u> Continuum.

$\mathbb{R}, [0, 1]$.

Example

$\mathbb{R}^2$

$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$.

$= \{ 0, 1, -1, 2, -2, \dots \}$.

Theorem A countable union of countable sets are countable.

$A_1, A_2, A_3, \dots$ then $\bigcup_{i=1}^{\infty} A_i$ - countable.

countable

$$\begin{array}{lllll}
A_1: & a^1_1 & a^1_2 & a^1_3 & \dots \\
A_2: & a^2_1 & a^2_2 & a^2_3 & \dots \\
A_3: & a^3_1 & a^3_2 & a^3_3 & \dots \\
\vdots & & & &
\end{array}$$

Rational numbers (positive) $\mathbb{Q}^+ = \bigcup_{j=1}^{\infty} \bigcup_{i=1}^{\infty} \left\{ \frac{i}{j} \right\}$.

$$= \bigcup_{j=1}^{\infty} \Big(\underbrace{\bigcup_{i=1}^{\infty} \left\{ \frac{i}{j} \right\}}_{= A_j} \Big)$$

Probability.

Terminology.

Experiment. - An unpredictable process by which an observation is made with well defined outcomes.

Ex: Flipping, Rolling a die.

Outcomes: sample points, simple events.

Sample space: S, Ω - the set of all outcomes.

Event: A subset of outcomes.

Ex: that flip heads.

Ex: Flip a coin 3 times
A - the event that you get 2 heads.

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}.$$

$A = \{HHT, HTH, THH\}$.

Axioms of Probability.

P - probability measure.

P: Event. $\longrightarrow [0,1]$.

designates how likely the event is.

Axioms

① (Non-negativity). $P(A) \geq 0$, $A \subset S$.

② (Normality) $P(S) = 1$

③ (Countable Additivity).

Let $\overset{1}{A_1}, \overset{2}{A_2}, \overset{3}{A_3}, \dots$ countable collection of events, which are mutually exclusive.

$$A_i \cap A_j = \phi \qquad i \neq j$$

$$P(\underbrace{A_1 \cup A_2 \cup A_3 \cup \dots}_{\bigcup_{i=1}^{\infty} A_i}) = \sum_{i=1} P(A_i).$$

$\underline{Ex}$ $\quad A \subset B \qquad P(A) \leq P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$