Announcements

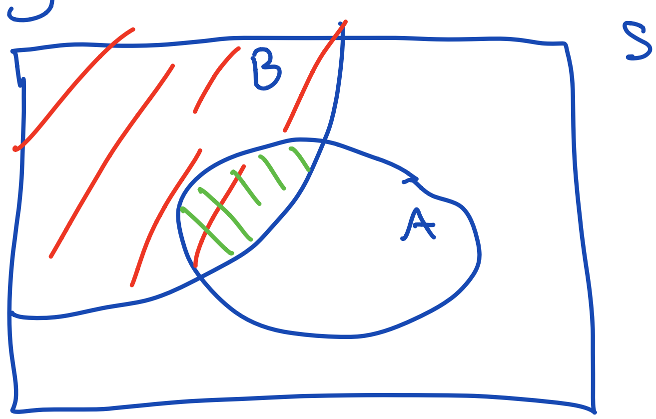
- Poll for conference sections on campuswire.
  ↳ please complete ASAP

- Course schedule updated.
- Assignments graded. (Regrade window available 24hrs).

Conditional Probability.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0.$$

- Probability of A given B.



$P(B \mid B) = 1 \longrightarrow$ normalized.

$A \longmapsto P(A \mid B)$ is a probability measure.

- $P(A|B) \geq 0$
- $P(B|B) = 1 = P(S|B) = 1$
- $P\left(\bigcup_{i=1}^{\infty} A_i \mid B\right) = \sum_{i=1}^{\infty} P(A_i|B)$. $A_i \cap A_j = \emptyset$ $i \neq j$.

Law of multiplication

A, B , $P(A), P(B) \neq 0$

$$P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)$$

Ex 52 card deck.
- Draw two cards
- What is probability of two spades?

A_1 = the first card is spade.
A_2 = the second card is a spade.

$$P(A_1 \cap A_2) = P(A_2|A_1)\,P(A_1)$$

$$P(A_1) = \frac{13}{52} = \frac{\#\text{ of spades}}{\#\text{ of cards}}$$

$$P(A_2|A_1) = \frac{12}{51} = \frac{\#\text{ of spades left}}{\#\text{ of cards left}}.$$

$$P(A_1 \cap A_2) = \left(\frac{12}{51}\right)\left(\frac{13}{52}\right) = \frac{1}{17}.$$

Later

$$P(A_1 \cap A_2) = \frac{|A_1 \cap A_2|}{|S|}.$$

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3 events A, B, C, $P(A), P(B), P(C) \neq 0$

$$P(A \cap B \cap C) = P(A \mid B \cap C)\, P(B \cap C)$$
$$= P(A \mid B \cap C)\, P(B \mid C)\, P(C).$$
$$\overset{\shortparallel}{P(A \mid B, C)}$$

General Formula

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1 \mid A_2 \cap A_3 \dots \cap A_n)$$
$$\times P(A_2 \mid A_3 \cap A_4 \dots \cap A_n)$$
$$\vdots$$
$$\times P(A_{n-1} \mid A_n)\, P(A_n)$$

Law of total probability.

B_1, B_2, B_3 partition S.

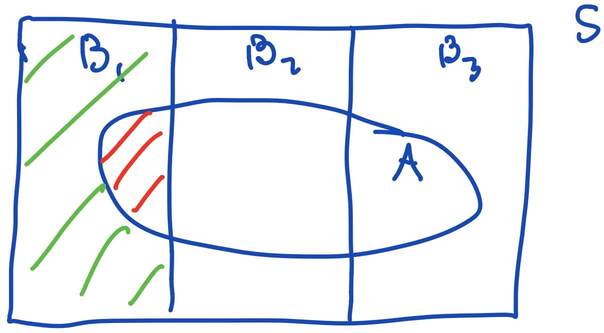
- $B_1 \cup B_2 \cup B_3 = S$, $\underset{i \neq j}{B_i \cap B_j = \phi}$

Split up A.

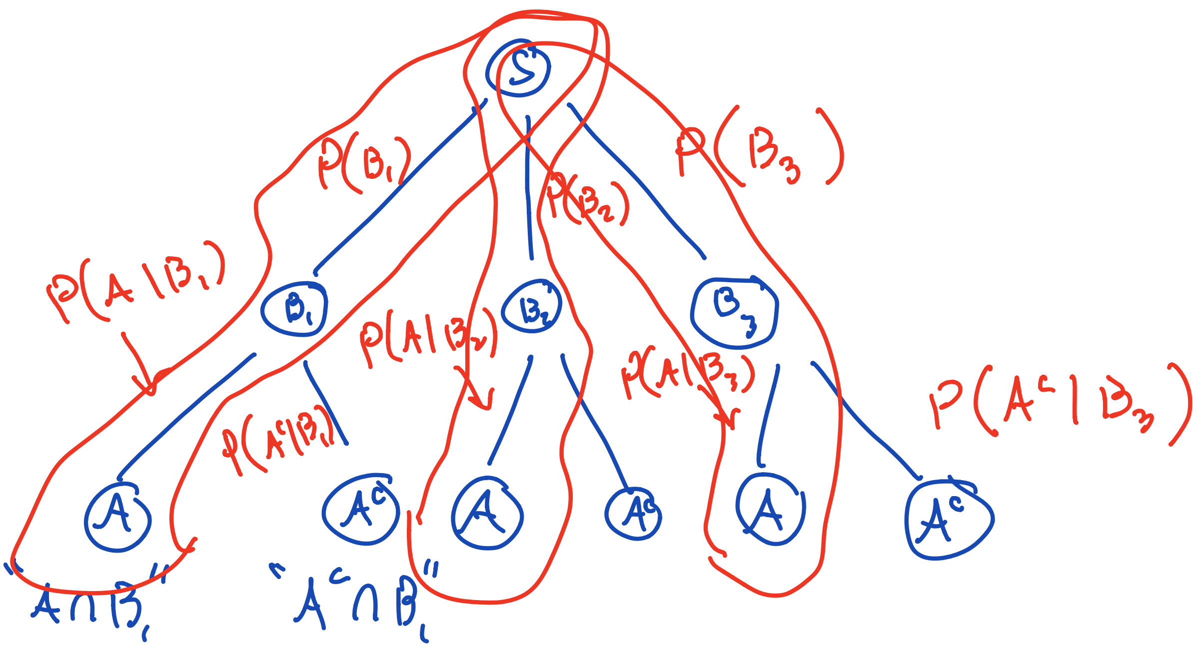
$$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \quad \text{- disjoint}$$

$$P(A) = P(A \cap B_1) + P(A \cup B_2) + P(A \cap B_3)$$
$$= P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + P(A \mid B_3)P(B_3)$$

law of total probability



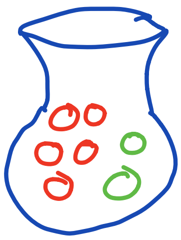
Probability Tree



$$P(A) = P(A \mid B_1)P(B_2) + P(A \mid B_2)P(B_3)$$

$+ P(A \mid B_3)P(B_3)$

Ex **Urn problem.**

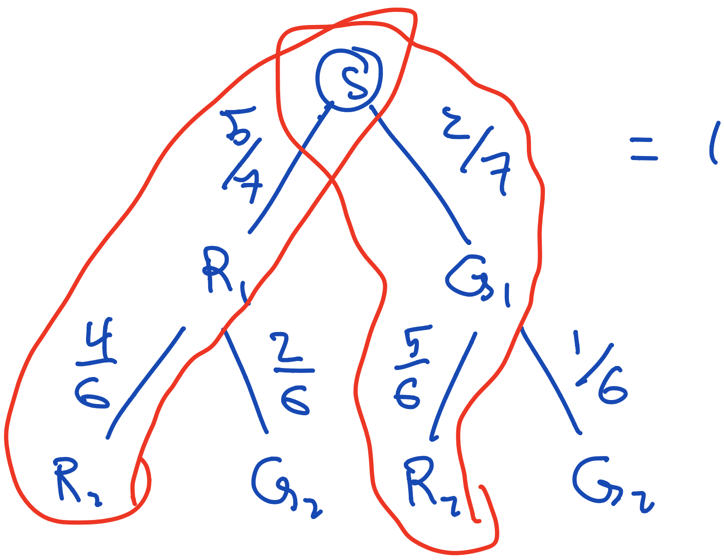


- Reach in pick out two balls
- no replacement.

What is the probability the second ball is red?

R_1 - first ball red $\qquad$ G_1 - first ball green

R_2 - second ball red $\qquad$ G_2 - second ball green.



$$P(R_2) = \left(\tfrac{4}{6}\right)\left(\tfrac{5}{7}\right) + \left(\tfrac{5}{6}\right)\left(\tfrac{2}{7}\right) = \tfrac{5}{7}$$

Law of total probability (General formula)

- Let $B_1, B_2, \ldots, B_n$ be a partition.

then

$$P(A) = \sum_{i=1} P(A \mid B_i) P(B_i)$$

Independence

Sometimes knowledge of one event doesn't change the probability of another.

Ex: Flip a coin twice.

Definition: A, B are independent if

$$P(A \cap B) = P(A) P(B).$$