Announcements

- Sign up for appropriate conference section by Friday.
  → See campuswire post.

Discrete Probability. (S - countable)

$$P(A) = \sum_{a \in A} P(\{a\})$$

sum over elementary outcomes.

Sample point method.

① List out the sample space. $S = \{s_1, s_2, \dots\}$

② Assign probabilities to elementary outcomes

find $P(\{s_i\})$ such that $\sum_{i=1}^{\infty} P(\{s_i\}) = 1$

③ Given a set A, use formula.

$$P(A) = \sum_{a \in A} P(\{a\})$$

Equally likely events ($|S| < \infty$).

- if all of the elementary outcomes have the same probability.
- $P(A) = \dfrac{|A|}{|S|}$ ✓ counting needed

Ex Rolling 2 dice for Monopoly.

- $S = \{2, 3, 4, 5, 6, \dots, 12\}$.

or ↳ sum of the outcomes ↳ not equally likely.

- $S = \{(1,1), (1,2), (1,3), \dots, (6,6)\}$.

↳ equally likely.

How many are in S?

$$\begin{array}{cccccc}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6). \\
(2,1) & (2,2) & (2,3) & - & - & \vdots \\
(3,1) & \dots & & & & \vdots \\
(4,1) & & & & & \vdots \\
(5,1) & & & & & \vdots \\
(6,1) & - & - & - & - & (6,6)
\end{array}$$

$|S| = 6 \times 6 = 36.$

Sum	2	3	4	5	6	7	8	9	10	11	12
Prob	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$\frac{5}{36}$	$4/36$	$3/36$	$2/36$	$1/36$

<u>Combinatorics</u> (theory of counting).

The foundation of combinatorics is one simple rule.

<u>The multiplication rule</u>

- Suppose I execute k sequential actions with

  n_1 ways to do action 1
  n_2 ways to do action 2
  $\vdots$
  n_k ways to do action k

Then there are

$$n_1 \times n_2 \times n_3 \times \cdots \times n_k$$

ways to perform these actions in sequence.

Ex Toss a die 3 times.

$$6 \times 6 \times 6 = 6^3 \quad \text{outcomes.}$$

Ex Olympics.

- 5 competitors in an event.
- How many ways can you assign Gold, Silver, Bronze to the competitors?

$$\underset{\text{assign gold}}{5} \times \underset{\text{assign silver}}{4} \times \underset{\text{assign bronze}}{3} = 60.$$

Terminology.

Sampling: Choosing an object from a set $\{a_1, a_2, \dots, a_n\}$.

We "draw" a sample from a set.

w or w/o replacement: Sample multiple times and either replace the element back in the set or leave it out.

Ordered / un-ordered: If order of the successive sampling matters or not.

Ordered sampling w/ replacement.

$A = \{1, 2, 3\}$, $k = 2$, sample twice.

$\uparrow n = 3$

order matters $(1,2) \neq (2,1)$

$$
\begin{array}{cccc}
 & & 3 & \\
 & (1,1), & (1,2) & (1,3) \\
3 & (2,1), & (2,2) & (2,3) \\
 & (3,1), & (3,2) & (3,3)
\end{array}
= 3 \times 3 = 9.
$$

General: $A = \{1, 2, 3, \ldots, n\}$.

$$\overbrace{\square \quad \square \quad \square \quad \cdots \quad \square}^{k}$$

$$n \times n \times n \times \cdots \times n = n^k$$

ways to sample k times w/ replacement.

- Ordered w/o replacement.

$A = \{1, 2, 3\}$, sample twice.

2

Cant choose $(1,1), (2,2), (3,3)$.

3

$(1,2), (1,3)$,
$(2,3), (2,1)$,
$(3,1), (3,2)$

3×2 ways.

Generally:

$A = \{1, 2, \ldots, n\}$, k samples.

$\square \quad \square \quad \square \quad \cdots \quad \square$ ~ k boxes.

$n \times (n-1) \times (n-2) \times \cdots \times (n-(k-1))$

ways to sample k things out of n w/o replacement

Known as the number of permutations.

$$P^n_k \equiv n(n-1)(n-2) \cdots (n-(k-1))$$
$$= \frac{n!}{(n-k)!}$$

Recall: factorial $n! = n(n-1)(n-2) \cdots 1$
$0! = 1$.

This is the number of ways to permute k things out of n distinct objects (where order matters)

<u>Ex</u> <u>Birthday problem</u>

<u>Question</u>: What is the probability that out of $\boxed{20}$ people at least 2 share the same birthday?

correction.

Ans: $= .41$ (Surprising?)

We will work this out next time.