Math 241 Exam 1 Fall 2014   **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) If $\bar{a} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 1\,\hat{k}$ and $\bar{b} = -4\,\hat{\imath} + 0\,\hat{\jmath} - 2\,\hat{k}$, find the sine and cosine of the angle θ between   [10 pts]
        $\bar{a}$ and $\bar{b}$.

    (b) Do the points $(0, 0, 0)$, $(0, 3, 4)$ and $(5, 2, 1)$ form a right triangle? Justify.   [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the equation of the plane containing the line $\frac{x}{2} = \frac{2-y}{3} = z$ and the point $(2, -1, 3)$ and
        check whether this plane contains the origin.   [10 pts]

    (b) Provide a piecewise smooth parametrization of the triangle with vertices $(0, 0)$, $(5, 2)$ and   [10 pts]
        $(4, 7)$.

Please put problem 3 on answer sheet 3

3. (a) Find the tangent and normal vectors for the curve $\bar{r}(t) = t^2\,\hat{\imath} + t\,\hat{\jmath}$ at $t = \sqrt{2}$.   [10 pts]

    (b) Find the value of a so that the curve $\bar{r}(t) = t\,\hat{\imath} + (2 + 3t)\,\hat{\jmath}$ for $0 \le t \le a$ has length 7.   [10 pts]

Please put problem 4 on answer sheet 4

4. (a) Sketch the sphere $x^2 + y^2 - 6y = -z^2 + 2z$. Mark at least two points with their coordinates.   [5 pts]

    (b) Find the equation of the sphere with center $(1, -2, 1)$ and which also contains the origin.   [5 pts]

    (c) Find the distance between the plane containing $(0, 0, 1)$, $(1, 2, 3)$ and $(0, -2, 4)$ and the origin.   [10 pts]

Please put problem 5 on answer sheet 5

5. (a) Sketch $\bar{r}(t) = 2\sin(t)\,\hat{\imath} + 5\cos(t)\,\hat{\jmath} - 1\,\hat{k}$ for $0 \le t \le \frac{\pi}{2}$. Label the start and end points with   [10 pts]
        their coordinates and indicate direction.

    (b) Let $\mathcal{L}$ be the line $\bar{r}(t) = (2 - t)\,\hat{\imath} + t\,\hat{\jmath} + (3t + 1)\,\hat{k}$ and let $\mathcal{P}$ be the plane $x + y + 2z = 10$.

        i. Find the point where $\mathcal{L}$ and $\mathcal{P}$ meet.   [5 pts]

        ii. Show that the line is not perpendicular to the plane.   [5 pts]

The End