Math 241 Exam 3 Spring 2015 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Note that only problem 4(b) requires you to draw a picture, however for other problems drawing pictures can certainly help you earn partial credit if something goes wrong.

Please put problem 1 on answer sheet 1

1. (a) Evaluate the following integral. This is the only integral you need to evaluate: [8 pts]

$$\int_0^2 \int_x^{x^2} xy \; dy \; dx$$

(b) Reparametrize the following integral as polar but do not evaluate: [12 pts]

$$\int_0^{\sqrt{2}/2} \int_y^{\sqrt{1-y^2}} \cos(x^2+y^2) \; dx \; dy$$

Please put problem 2 on answer sheet 2

2. (a) Let R be the region inside the circle $r = \sin\theta$ and outside the circle $r = \sqrt{3}/2$. Set up the iterated double integral in polar coordinates for $\iint_R \sqrt{x^2+y^2} \; dA$. Do not evaluate. [12 pts]

(b) Set up the system of equations you would need to solve in order to find the minimum and maximum of the function $f(x,y) = x^2y + x$ subject to the constraint $x^2 + y^4 = 16$ using Lagrange Multipliers. Do not go any further than giving the system of equations. [8 pts]

Please put problem 3 on answer sheet 3

3. (a) Let D be the region inside the cylinder $(x-1)^2 + y^2 = 1$ and between the planes $z = 0$ and $z = 10 - x$. Write down the iterated triple integral in cylindrical coordinates for the volume of D. Do not evaluate. [12 pts]

(b) Let D be the solid in the first octant, between the spheres $x^2+y^2+z^2 = 4$ and $x^2+y^2+z^2 = 9$ and below the cone $z = \sqrt{3x^2+3y^2}$. If the density of D is given by $f(x,y,z) = xz$, write down the interated triple integral in spherical coordinate for the mass of D. Do not evaluate. [10 pts]

Please put problem 4 on answer sheet 4

4. (a) Write down a parametrization of the cylinder $x^2 + z^2 = 9$ between $y = -1$ and $y = 3$. [8 pts]

(b) Draw a reasonable sketch of the surface parametrized by $\bar{r}(r,\theta) = r\cos\theta\,\hat{\imath} - 1\,\hat{\jmath} + r\sin\theta\,\hat{k}$ for $0 \le r \le 2$ and $0 \le \theta \le \pi$. Make sure your sketch has some sense of scale and position. [8 pts]

Please put problem 5 on answer sheet 5

5. Let R be the region in the first quadrant bounded by the lines $y = 1/x$, $y = 3/x$, $y = 2x$ and $y = 4x$. Perform a change of variables to rewrite $\iint_R xy \; dA$ as an interated double integral over a rectangle in the uv-plane. Do not evaluate. [22 pts]

The End