Math 241 Exam 4 Fall 2014 **Justin Wyss-Gallifent**

Directions: Carefully read the directions as to whether to evaluate your integrals. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. Let Σ be the portion of $z = 16 - x^2 - y^2$ inside the cylinder $r = 2\cos\theta$ and with upwards orientation. Draw a picture of Σ and find the rate at which the fluid $\bar{F}(x,y,z) = 0\,\hat{\imath} + x\,\hat{\jmath} + 0\,\hat{k}$ is flowing through Σ. [20 pts]
   Stop when you have an iterated double integral.

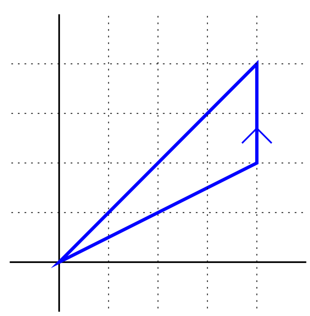
Please put problem 2 on answer sheet 2

2. (a) Evaluate $\int_C y\ dx + (x+1)\ dy$ where C is parametrized by $\bar{r}(t) = e^t \sin(\pi t)\,\hat{\imath} + e^t \cos(\pi t)\,\hat{\jmath}$ for $0 \le t \le \frac{1}{2}$. [7 pts]
   Stop when you have an unsimplified numerical answer.

   (b) Find the mass of the wire C, where C is the line segment in the xy-plane joining $(2,0)$ to $(5,4)$ and the density is $f(x,y) = 3xy$. [13 pts]
   Stop when you have an unsimplified numerical answer.

Please put problem 3 on answer sheet 3

3. Evaluate $\int_C x^2\ dx + 3xy\ dy$ where C is the curve shown in the picture. [20 pts]
   Stop when you have an unsimplified numerical answer.



Please put problem 4 on answer sheet 4

4. Let C be the triangle with vertices $(5,0,0)$, $(0,5,0)$ and $(0,0,5)$ oriented counterclockwise when viewed from above. Use Stokes' Theorem to find the work done on a particle by the force $\bar{F}(x,y,z) = yz\,\hat{\imath} + y\,\hat{\jmath} + xy\,\hat{k}$ as the particle traverses the curve C. Include a picture of C and Σ (these can be together on one picture). [25 pts]
   Stop when you have an iterated double integral.

Please put problem 5 on answer sheet 5

5. Let Σ be the portion of the cone $z = \sqrt{x^2 + y^2}$ inside the sphere $x^2 + y^2 + z^2 = 9$ as well as the portion of the sphere inside the cone. [15 pts]
   Find the rate at which the fluid $\bar{F}(x,y,z) = y\,\hat{\imath} + x\,\hat{\jmath} + z^2\,\hat{k}$ is flowing inwards through Σ.
   Stop when you have an iterated triple integral.

The End!