Math 241 Exam 4 Spring 2015 **Dr. Justin O. Wyss-Gallifent**

Directions: Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem. Do not evaluate integrals or simplify numerical answers unless explicitly stated.

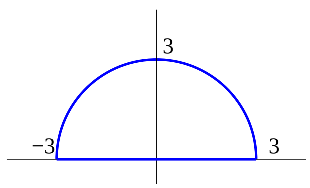
Please put problem 1 on answer sheet 1

1. (a) Find the mass of the wire segment C (evaluate!) parametrized by $\mathbf{r}(t) = t\,\mathbf{i} + (3t+1)\,\mathbf{j}$ for $0 \le t \le 4$ if the density is given by $f(x,y) = xy$. [10 pts]

   (b) Evaluate $\int_C 2y\ dx + (2x+z)\ dy + y\ dz$ where C is the curve parametrized by $\mathbf{r}(t) = (t^2+1)\,\mathbf{i} + \sqrt{t}\,\mathbf{j} - 4t\,\mathbf{k}$ for $1 \le t \le 4$. [10 pts]

Please put problem 2 on answer sheet 2

2. Evaluate $\int_C 2y^2\ dx + 3y\ dy$ where C is the semicircle shown with clockwise orientation: [20 pts]
   You must evaluate this integral!



Please put problem 3 on answer sheet 3

3. Suppose Σ is the surface consisting of the portion of the paraboloid $z = x^2 + y^2$ above the filled-in triangle in the xy-plane with corners $(0,0,0)$, $(0,6,0)$ and $(6,0,0)$. If the density is given by $f(x,y,z) = yz$ find an iterated double integral which gives the total mass of Σ but do not evaluate. [20 pts]

Please put problem 4 on answer sheet 4

4. Let Σ be the portion of the plane $x+y=2$ in the first octant between $z=0$ and $z=3$. Let C be its edge, oriented counterclockwise when looking in towards the octant. Apply Stokes' Theorem to the line integral [20 pts]

$$\int_C (x\,\mathbf{i} + xy\,\mathbf{j} + xyz\,\mathbf{k}) \cdot d\mathbf{r}$$

   Parametrize the resulting surface and proceed until you have an iterated double integral but do not evaluate.

Please put problem 5 on answer sheet 5

5. Suppose Σ is the portion of the cylinder $x^2+y^2=9$ between $z=1$ and $z=5$ along with the disks which seal it off at each end. Assume Σ is oriented inwards. Use the Divergence Theorem to evaluate the following integral: [20 pts]

$$\iint_\Sigma (x\,\mathbf{i} + 2y\,\mathbf{j} + z^2\,\mathbf{k}) \cdot \mathbf{n}\ dS$$

The End!