Math 241 Fall 2012 Final Exam

- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets.
- No calculators are permitted.
- One page of notes is permitted.
- Do not evaluate integrals or simplify answers unless indicated.

Please put problem 1 on answer sheet 1

1. Consider the vectors $\bar{a} = 2\,\hat{\imath} - \hat{\jmath} + 3\,\hat{k}$ and $\bar{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$.

   (a) Find the projection of $\bar{a}$ onto $\bar{b}$. Call this $\bar{p}$. [7 pts]

   (b) Find a vector $\bar{c}$ so that $\bar{a} = \bar{p} + \bar{c}$. [5 pts]

   (c) Are $\bar{c}$ and $\bar{p}$ orthogonal (perpendicular)? Justify. [5 pts]

   (d) Find the equation of the plane containing the point $(1, 2, 3)$ which is parallel to both the vectors $\bar{a}$ and $\bar{b}$. [8 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the length of the curve parametrized by [15 pts]

$$\bar{r}(t) = \frac{2\sqrt{2}}{3}t^{3/2}\,\hat{\imath} + t\sin t\,\hat{\jmath} + t\cos t\,\hat{k} \text{ for } 0 \le t \le \pi$$

   Note: If you're careful the integrand should simplify a lot before integration.

   (b) Find the tangential component of acceleration for $\bar{r}(t) = e^t\,\hat{\imath} + t\sqrt{2}\,\hat{\jmath} + e^{-t}\,\hat{k}$. [10 pts]

Please put problem 3 on answer sheet 3

3. Let $f(x, y, z) = xe^{(y^2 - z^2)}$.

   (a) Find the direction of maximum increase of f at the point $(1, 2, -2)$. [10 pts]

   (b) Find the directional derivative of f in the direction of motion of the object parametrized by $\bar{r}(t) = t\,\hat{\imath} + 2\cos(t-1)\,\hat{\jmath} - 2e^{t-1}\,\hat{k}$ at $(1, 2, -2)$. [15 pts]

Please put problem 4 on answer sheet 4

4. Use the method of Lagrange multipliers to find the maximum and minimum of the function $f(x, y) = x^2 + (y-2)^2$ on the hyperbola $x^2 - y^2 = 1$. [25 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Evaluate the integral [25 pts]

$$\iint_R y \, dA$$

where R is the region in the xy-plane bounded above by the circle $x^2 + y^2 = 2$ and below by the parabola $y = \frac{1}{2}x^2 - 1$.

Please put problem 6 on answer sheet 6

6. Consider the solid D inside the cylinder $x^2 + y^2 = 4$, bounded below by the xy-plane and above by the cone $z = \sqrt{x^2 + y^2}$. Consider the triple integral

$$\iiint_D (x^2 + y^2)^{3/2} \, dV$$

   (a) Rewrite this as an iterated integral in cylindrical coordinates. [15 pts]

   (b) Evaluate this integral. [10 pts]

Please put problem 7 on answer sheet 7

7. Let C be the intersection of the paraboloid $z = 9 - x^2 - y^2$ and the cylinder $r = 3\sin\theta$ with clockwise orientation when viewed from above. Use Stokes' Theorem to convert [25 pts]

$$\int_C (2xy\,\hat{\imath} + z\,\hat{\jmath} - xz\,\hat{k}) \cdot d\bar{r}$$

to a surface integral and then parametrize the surface and proceed to an iterated double integral but do not evaluate.

Please put problem 8 on answer sheet 8

8. (a) Evaluate the line integral $\int_C -z \, ds$ where C is the curve parametrized by [15 pts]

   $\bar{r}(t) = \frac{2}{3}t^{3/2}\,\hat{\imath} - 2t\,\hat{\jmath} - 6\,\hat{k}$ for $0 \le t \le 3$.

   (b) Suppose Σ is composed of both the hemisphere $z = \sqrt{9 - x^2 - y^2}$ and the disk $x^2 + y^2 \le 9$ in the xy-plane, oriented outwards. Evaluate [10 pts]

$$\iint_\Sigma \left[(2x - y)\,\hat{\imath} + (3xy + y)\,\hat{\jmath} + (z - 3xz)\,\hat{k}\right] \cdot \bar{n} \, dS$$

Welcome to the End of the Exam