Math 241 Fall 2014 Final Exam

- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- Do not evaluate integrals or simplify answers unless indicated.

Please put problem 1 on answer sheet 1

1. Given the line $\mathcal{L}$ with symmetric equation $x = \frac{y-1}{3} = \frac{z}{2}$, the plane $\mathcal{P}$ with equation $9x-2y-z = 0$ and the point $\mathcal{Q} = (1, -2, 5)$:
   (a) Determine whether the line $\mathcal{L}$ is parallel to the plane $\mathcal{P}$. [10 pts]
   (b) Find the distance from the point $\mathcal{Q}$ to the line $\mathcal{L}$. Simplify. [15 pts]

Please put problem 2 on answer sheet 2

2. Let the position of an object in motion be given by $\mathbf{r}(t) = e^t \cos t\, \mathbf{i} + e^t \sin t\, \mathbf{j} + e^t\, \mathbf{k}$.
   (a) Find the velocity and acceleration of the object at any t. [10 pts]
   (b) Write down the integral for the distance traveled by the object between $t = 0$ and $t = 2$ but do not evaluate. [5 pts]
   (c) Compute the curvature of the object's path at $t = 0$. [10 pts]

Please put problem 3 on answer sheet 3

3. Use the method of Lagrange Multipliers to determine the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $4x^2 + y^2 = 4$. You may assume that the maximum and minimum exist. [25 pts]

Please put problem 4 on answer sheet 4

4. (a) Let $D(x, y) = 300 - 2x^2 - 3y^2$ denote the depth of a lake in feet. If a boat is at $(3, 5)$, in what direction should the boat travel for the depth of the water to increase most rapidly and what would that rate of increase be? [10 pts]
   (b) Ohm's Law states that $I = \frac{V}{R}$ which relates current (I) with voltage (V) and resistance (R). Suppose the voltage is decreasing at 5 volts/second while the resistance is decreasing at 2 ohms/second. Find the rate of change of the current with respect to time when the voltage is 80 volts and the resistance is 40 ohms. [15 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Let R be the region in the xy-plane above the graph of $y = x^2$ and below the graph of $y = -(x-1)^2 + 1$. Let D be the solid above R and below the plane $x + y + z = 5$.

   (a) Separately sketch reasonable pictures of both R and D. [10 pts]

   (b) Set up an iterated double integral for the volume of D. Do not evaluate. [15 pts]

Please put problem 6 on answer sheet 6

6. Let R be the parallelogram in the xy-plane formed by the lines $x + y = 1$, $x + y = 2$, $2y - x = 2$ and $2y - x = 0$.

   (a) Sketch R. [5 pts]

   (b) Use a change of variables to evaluate $\iint_R x + y \; dA$. Make sure to draw the new region in the uv-plane. This integral must be evaluated! [20 pts]

Please put problem 7 on answer sheet 7

7. Let C be the edge of the part of the plane $2x + 2y + z = 10$ in the first octant, oriented counterclockwise when viewed from above.

   (a) Apply Stokes' Theorem to the integral $\int_C 2y \; dx + x \; dy + xz \; dz$ to get a surface integral over a surface Σ. Describe Σ, including its induced orientation. Either words or a picture suffice. [8 pts]

   (b) Parametrize Σ and convert your answer to (a) to an iterated double integral. [12 pts]

   (c) Evaluate. [5 pts]

Please put problem 8 on answer sheet 8

8. (a) Let C be the part of the graph of the function $y = x^2$ from $x = 1$ to $x = 2$. Write down the iterated single integral corresponding to $\int_C x - y \; ds$. Do not evaluate. [10 pts]

   (b) Let D be the solid inside the cone with spherical equation $\phi = \frac{\pi}{6}$ and below the plane $z = 3$. Let Σ be the surface of D oriented inwards. Apply the Divergence Theorem to the integral $\iint_\Sigma (x\,\mathbf{i} + xz\,\mathbf{j} + z^2\,\mathbf{k}) \cdot \mathbf{n} \; dS$ and then use a spherical parametrization to obtain an iterated triple integral. Do not evaluate. [15 pts]

Welcome to the End of the Exam