Math 241 Fall 2015 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*
- Numerical answers do not need to be simplified.

Please put problem 1 on answer sheet 1

1. (a) Find the point where the line with parametric equations $x = 1 + 2t$, $y = 2 - t$, $z = 4 - 2t$ meets the plane $x + y - 2z = 10$. [10 pts]

   (b) Find the symmetric equations of the line perpendicular to the plane $x + y - 2z = 10$ and passing through the point $(1, 2, 3)$. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find an equation of the plane passing through the points $(-2, 1, 1)$, $(0, 2, 3)$ and $(1, 0, -1)$. [10 pts]

   (b) Let $\mathbf{a} = 2\,\mathbf{i} + 3\,\mathbf{j} - 1\,\mathbf{k}$ and $\mathbf{b} = 0\,\mathbf{i} + 6\,\mathbf{j} + 10\,\mathbf{k}$. Find $\mathbf{pr_a b}$. [10 pts]

Please put problem 3 on answer sheet 3

3. Suppose C is parametrized by $\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\,\mathbf{j}$ for $0 \le t \le 3$.

   (a) Find the length of the curve C. Simplify. [10 pts]

   (b) Find the unit tangent vector $\mathbf{T}(t)$ and unit normal vector $\mathbf{N}(t)$. Simplify. [10 pts]

Please put problem 4 on answer sheet 4

4. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C (2xy + z)\ dx + x^2\ dy + x\ dz$ where C is parametrized by $\mathbf{r}(t) = 16t^2\,\mathbf{i} + \frac{1}{t}\,\mathbf{j} + (2t - 1)\,\mathbf{k}$ for $\frac{1}{2} \le t \le 1$. [20 pts]

Please put problem 5 on answer sheet 5

5. The object distance $x > 0$, image distance $y > 0$ and focal length L of a simple lens satisfy: [20 pts]

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{L}$$

   Using Lagrange multipliers find the minimum of $f(x, y) = x + y$ subject to the constraint above. You may assume that the minimum exists and that L is a fixed constant.

Turn Over!

Please put problem 6 on answer sheet 6

6. (a) Let $z(x, y) = x^2 - xy^2$. For all (x, y) compute [10pts]

$$\frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y} \right)^2$$

(b) By differentiating both sides of the equation [10pts]

$$f(tx, ty) = t^2 f(x, y)$$

with respect to t and then setting $t = 1$, show that

$$x \frac{\partial f(x, y)}{\partial x} + y \frac{\partial f(x, y)}{\partial y} = 2f(x, y)$$

Please put problem 7 on answer sheet 7

7. Let R be the region in the $xy-$plane between the graphs of $y = x^2$ and $y = 1 - x^2$. Let D be the solid region between R and the parabolic sheet $z = x^2$. Find the volume of D. Simplify as much as possible. [20 pts]

Please put problem 8 on answer sheet 8

8. Let D be the solid region inside the sphere $\rho = 2$ and inside the cone $z = \sqrt{x^2 + y^2}$. Evaluate the integral $\iiint_D z^2 \; dV$ using spherical coordinates. [20 pts]

Please put problem 9 on answer sheet 9

9. Let C be the intersection of the cylinder $x^2 + z^2 = 4$ with the plane $x + y = 4$ and with counterclockwise orientation when viewed from the positive y-axis. Use Stokes' Theorem to convert the line integral [20 pts]

$$\int_C xy \; dx + y \; dy + xz \; dz$$

to a surface integral. Write your integral as an iterated integral in polar coordinates. Do not evaluate this integral.

Please put problem 10 on answer sheet 10

10. (a) Let C be the triangle in the xy-plane with corners $(0, 0)$, $(4, 2)$ and $(0, 6)$, oriented clockwise. By using Green's Theorem calculate the line integral $\int_C 3xy \; dx + 4x^2 \; dy$. [10 pts]

(b) Evaluate $\int_C x^2 y \; ds$ where C is the circle $x^2 + y^2 = 4$. [10 pts]

Welcome to the End of the Exam