Math 241 Spring 2011 Final Exam

Directions: Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets. No calculators or notes are permitted for this exam. Integrals must be evaluated unless otherwise indicated. Numerical answers do not need to be simplified.

Please put problem 1 on answer sheet 1

1. (a) Find the equation of the plane containing both the point $(3,0,0)$ and the line [7 pts]

$$x-1=\frac{1-y}{2}=z-1$$

   (b) Find the distance from the point $(0,0,0)$ to the plane you found in (a). [6 pts]

   (c) Compute the gradient of $f(x,y,z)=\sqrt{xy^2z^3}$ at $(2,2,2)$. [6 pts]

   (d) Compute the directional derivative of $f(x)$ at $(2,2,2)$ in the direction of $\hat{\imath}+\hat{\jmath}+\hat{k}$. [6 pts]

Please put problem 2 on answer sheet 2

2. Find all critical points of the function [25 pts]

$$f(x,y)=x^3y-3xy+y^2$$

   and characterize each as local maximum, local minimum or saddle point.

Please put problem 3 on answer sheet 3

3. Compute the arc length of the curve [25 pts]

$$\bar{r}(t)=(5t-\sin(5t))\hat{\imath}+(5-\cos(5t))\hat{\jmath} \quad \text{with} \quad 0\le t\le 2\pi/5$$

Please put problem 4 on answer sheet 4

4. Compute the integral [25 pts]

$$\iiint_D z\ dV$$

   where D is the solid inside the sphere $x^2+y^2+z^2=25$ and in the first octant.

Please put problem 5 on answer sheet 5

5. (a) Compute the integral [10 pts]

$$\int_C yz\ dx+xz\ dy+xy\ dz$$

   where C is the curve

$$\bar{r}(t)=2\cos t\hat{\imath}+2\sin t\hat{\jmath}+3t\hat{k} \quad \text{with} \quad 0\le t\le \pi$$

   (b) Use Green's Theorem to calculate the integral [15 pts]

$$\int_C (x^2+y^2)\ dx-2xy\ dy$$

   where C is the triangle with vertices $(0,0)$, $(2,0)$ and $(0,1)$ with counterclockwise orientation.

Turn Over

Please put problem 6 on answer sheet 6

6. Consider $\iint_\Sigma \bar{F}\cdot\bar{n}\ dS$ where $\bar{F} = y\hat{\imath}+xy\hat{\jmath}+xz\hat{k}$ and Σ is the portion of the plane $x+2y+2z = 10$ in the first octant oriented away from the origin. Proceed until you have an iterated double integral in rectangular coordinates and then stop. Do not evaluate this integral. [25 pts]

Please put problem 7 on answer sheet 7

7. Let $\bar{F} = xy\hat{\imath}+x^2\hat{\jmath}+z^2\hat{k}$ and let C be the intersection of the cylinder $x^2+y^2 = 9$ with the plane $z = y$ and with counterclockwise orientiation when viewed from above. Use Stokes's Theorem to calculate [25 pts]

$$\int_C \bar{F}\cdot d\bar{r}$$

Please put problem 8 on answer sheet 8

8. Let $\bar{F} = \hat{\imath}+\hat{\jmath}+\hat{k}$ and $\bar{G} = x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ and let Σ be the sphere $x^2+y^2+z^2 = 25$ with outward orientation.

   (a) Show that $\nabla\cdot(\bar{F}\times\bar{G}) = 0$. [15 pts]

   (b) Use Gauss' formula (the Divergence Theorem) to calçulate [10 pts]

$$\iint_\Sigma (\bar{F}\times\bar{G})\cdot\bar{n}\ dS$$

The End