Math 241 Spring 2014 Final Exam

- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets.
- No calculators are permitted.
- Do not evaluate integrals or simplify answers unless indicated.

Please put problem 1 on answer sheet 1

1. (a) Find the symmetric equation of the line containing $(1,2,3)$ and $(-1,5,3)$. [10 pts]
   (b) Find the distance between $(3,-5,2)$ and the plane $2x-y+3z=6$. Simplify. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) For $\mathbf{u}=2\,\mathbf{i}-\mathbf{j}+3\,\mathbf{k}$ and $\mathbf{v}=4\,\mathbf{i}+\mathbf{j}-2\,\mathbf{k}$, find $Pr_{\mathbf{u}}\mathbf{v}$. [10 pts]
   (b) Find the curvature $\kappa(1)$ of $\mathbf{r}(t)=t^2\,\mathbf{i}+t^3\,\mathbf{j}$. [10 pts]

Please put problem 3 on answer sheet 3

3. (a) Find $\mathbf{T}(1)$ for $\mathbf{r}(t)=t\,\mathbf{i}-2t^3\,\mathbf{j}+\frac{1}{t}\,\mathbf{k}$. [5 pts]
   (b) Find the tangential component of acceleration for $\mathbf{r}(t)=t^3\,\mathbf{i}-4t\,\mathbf{j}+t^2\,\mathbf{k}$ at $t=2$. [5 pts]
   (c) Find the point at which the line $\mathbf{r}(t)=(t+1)\,\mathbf{i}-2t\,\mathbf{j}+(3t-2)\,\mathbf{k}$ passes through the plane $x+y-z=10$. [10 pts]

Please put problem 4 on answer sheet 4

4. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x,y)=xy$ on the circle $(x-2)^2+y^2=4$. [20 pts]

Please put problem 5 on answer sheet 5

5. Find and categorize all relative extrema for the function $f(x,y)=x^3-2xy+y^2$. [20 pts]

Turn Over!

Please put problem 6 on answer sheet 6

6. Let $f(x,y) = \ln(x^2 + xy + y^2)$.

   (a) Find the direction of maximum increase of f at $(1,0)$ as a unit vector. [7 pts]

   (b) Find the maximum directional derivative at $(1,0)$. [6 pts]

   (c) Calculate the directional derivative of f at $(0,1)$ in the direction of $2\,\mathbf{i} + 3\,\mathbf{j}$. [7 pts]

Please put problem 7 on answer sheet 7

7. (a) Find a parametrization for the part of the cylinder $y^2 + z^2 = 1$ which lies between $x = -2$ and $x = 2$. [5 pts]

   (b) Find the equation of the plane tangent to the cylinder in part (a) at the point $\left(1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Write your answer in the form $ax + by + cz = d$. [15 pts]

Please put problem 8 on answer sheet 8

8. Find the volume of the solid region D that is bounded on the sides by the upper nappe of the cone $z^2 = \frac{1}{3}(x^2 + y^2)$, on the top by the sphere $x^2 + y^2 + z^2 = 9$ and below by the sphere $x^2 + y^2 + z^2 = 1$. [20 pts]

Please put problem 9 on answer sheet 9

9. Let C be the intersection curve of the parabolic sheet $y = x^2$ with the cylinder $x^2 + z^2 = 4$, oriented clockwise when viewed from the positive y-axis. Apply Stokes' Theorem to the integral $\int_C 2y\,dx + xz\,dy + z^2\,dz$ and continue until you have an iterated double integral. Do not evaluate. [20 pts]

Please put problem 10 on answer sheet 10

10. (a) Evaluate $\int_C 7y\,dx + 12y\,dy$ where C is the semicircle $y = \sqrt{9 - x^2}$ along with the line segment joining $(-3,0)$ with $(3,0)$, oriented clockwise. [8 pts]

    (b) Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 4$ inside the cylinder $x^2 + y^2 - 2y = 0$ as an iterated double integral in r and θ. Do not evaluate. [12 pts]

Welcome to the End of the Exam