Math 241H - Section 0301 - Fall 2017 Name:____________________________
Quiz 10

You have 15 minutes to complete this quiz. No calculator, cheat sheet or aid of any kind is allowed.

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1. [10pts] Let C be the closed curve that lies in the intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$, oriented in the counter-clockwise direction when viewed from above. Apply Stoke's Theorem to the line integral

$$\int_C xz\mathrm{d}x + yz\mathrm{d}y - x^2\mathrm{d}z,$$

and write the resulting flux integral as an iterated integral (DO NOT EVALUATE).

Solution: Let $\mathbf{F} = xz\mathbf{i} + yz\mathbf{j} - x^2\mathbf{k}$. To apply Stokes Theorem we take the curl of $\mathbf{F}$,

$$\nabla \times \mathbf{F} = -y\mathbf{i} + 3x\mathbf{j}.$$

The surface Σ with boundary C has induced upward facing normal $\mathbf{n}$. It is the graph of a function $z = f(x, y) = x^2 - y^2$ for (x, y) belonging to the circular region R define by $x^2 + y^2 \leq 1$. We apply Stokes Theorem and then parameterize the resulting surface integral to obtain

$$\begin{aligned}
\int_C xz\mathrm{d}x + yz\mathrm{d}y - x^2\mathrm{d}z &= \iint_\Sigma (-y\mathbf{i} + x\mathbf{j}) \cdot \mathbf{n}\mathrm{d}S \\
&= \iint_R (-y\mathbf{i} + 3x\mathbf{j}) \cdot (-2x\mathbf{i} + 2y\mathbf{j} + \mathbf{k})\,\mathrm{d}A \\
&= \iint_R 8xy\,\mathrm{d}A \\
&= \int_0^{2\pi} \int_0^1 8r^3 \sin\theta \cos\theta \mathrm{d}r\mathrm{d}\theta.
\end{aligned}$$