Math 241H - Section 0301 - Fall 2017 Name:_____
Quiz 2

You have 20 minutes to complete this quiz. No calculator, cheat sheet or aid of any kind is allowed.

1.[5pts] Show that the line

$$x - 1 = y + 4 = \frac{z}{2}$$

is parallel to the plane

$$3x - y - z = 0.$$

Solution. We begin by noting that the line will be parallel to the plane if the direction $\mathbf{L}$ that the line points in is perpendicular to the $\mathbf{N}$ the normal to the plane. Therefore after finding $\mathbf{L}$ and $\mathbf{N}$ if suffices to check that

$$\mathbf{L} \cdot \mathbf{N} = 0.$$

From the equation of the line we see that $\mathbf{L} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, and from the equation of the plane, $\mathbf{N} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$. We find that

$$\mathbf{L} \cdot \mathbf{N} = 1(3) - 1(1) + 2(-1) = 3 - 1 - 2 = 0.$$

Therefore the plane and the line are parallel.

2.[5pts] What is the distance between the line and the plane given in Problem 1? *(Hint: Find a point P_1 on the line and find the distance between P_1 and the plane.)*

Solution. Since the line is parallel to the plane, we can determine the distance from the line to the plane by finding the distance from any point on the line to the plane. The formula we will use is

$$D = \frac{|\mathbf{N} \cdot \overrightarrow{P_0P_1}|}{\|\mathbf{N}\|}$$

where P_0 is a point on the plane and P_1 is a point on the line. From the equations, we can immediately see that $P_0 = (0, 0, 0)$ is a point on the plane and $P_1 = (1, -4, 0)$ is a point on the line. It follows that

$$\mathbf{N} \cdot \overrightarrow{P_0P_1} = 3(1) - 1(-1) - 1(0) = 4,$$

and

$$\|\mathbf{N}\| = \sqrt{3^2 + (-1)^2 + (-1)^2} = \sqrt{11}.$$

Therefore

$$D = \frac{4}{\sqrt{11}}.$$