Math 241H - Section 0301 - Fall 2017 Name:____________________________
Quiz 3

You have 20 minutes to complete this quiz. No calculator, cheat sheet or aid of any kind is allowed.

1.[5pts] Consider the vector-valued function

$$\mathbf{r}(t) = 4\sin t\mathbf{i} - 3\sin t\mathbf{j} + 5\cos t\mathbf{k},$$

coresponding to circle in a plane. Compute the quantity

$$(\mathbf{r} \times \mathbf{r}')(t).$$

How does this value relate to the plane containing the circle parameterized by $\mathbf{r}(t)$?

Solution. We first find that

$$\mathbf{r}'(t) = 4\cos t\mathbf{i} - 3\cos t\mathbf{j} - 5\sin t\mathbf{k}.$$

Taking the cross product, we arive at

$$\begin{aligned}\mathbf{r}(t) \times \mathbf{r}'(t) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\sin t & -3\sin t & 5\cos t \\ 4\cos t & -3\cos t & -5\sin t \end{vmatrix} \\ &= 15(\sin^2 t + \cos^2 t)\mathbf{i} + 20(\sin^2 t + \cos^2 t)\mathbf{j} - 12(\sin t\cos t - \sin t\cos t)\mathbf{k}. \\ &= 15\mathbf{i} + 20\mathbf{j}.\end{aligned}$$

This value is the normal to the plane that the circular motion lies in.

2. [5pts] Let $\mathbf{r}(t)$ be the position of a particle with mass m and let $\mathbf{v}(t) = \mathbf{r}'(t)$ be it's velocity. Suppose that the particle is subject to a force $\mathbf{F}(t)$. Define the angular momentum $\mathbf{L}(t)$ and torque $\mathbf{T}(t)$ on the particle by

$$\mathbf{L}(t) = (\mathbf{r} \times m\mathbf{v})(t), \quad \mathbf{T}(t) = (\mathbf{r} \times \mathbf{F})(t).$$

Using Newton's law

$$m\mathbf{v}'(t) = \mathbf{F}(t),$$

derive the following *angular version* of Newton's law

$$\mathbf{L}'(t) = \mathbf{T}(t).$$

(Hint: take the derivative of $\mathbf{L}(t)$).

Solution. Taking the derivative of $\mathbf{L}(t)$ we obtain

$$\begin{aligned}
\mathbf{L}'(t) &= \mathbf{r}'(t) \times m\mathbf{v}(t) + \mathbf{r}(t) \times m\mathbf{v}'(t) \\
&= \underbrace{\mathbf{v}(t) \times m\mathbf{v}(t)}_{=0} + \mathbf{r}(t) \times \mathbf{F}(t) \\
&= \mathbf{T}(t).
\end{aligned}$$