Math 241H - Section 0301 - Fall 2017 Name:____________________________

Quiz 5

You have 15 minutes to complete this quiz. No calculator, cheat sheet or aid of any kind is allowed.

1.[5pts] Find the point $P_0 = (x_0, y_0, z_0)$ on the hyperbolic paraboloid

$$z = x^2 - 3y^2$$

so that the tangent plane at P_0 is parallel to the plane $8x + 3y - z = 4$.

Solution. The hyperbolic paraboloid is given by

$$z - x^2 + 3y^2 = 0.$$

The normal is given by the gradient

$$\mathbf{N} = \nabla(z - x^2 + 3y^2) = -2x\mathbf{i} + 6y\mathbf{j} + \mathbf{k}.$$

Additionally, the normal to the plane $8x + 3y - z = 4$ is

$$\mathbf{N}_1 = 8\mathbf{i} + 3\mathbf{j} - \mathbf{k}.$$

The planes are paralell if there is a c such that,

$$\mathbf{N} = c\mathbf{N}_1.$$

This implies that

$$-2x = 8c, \quad 6y = 3c, \quad 1 = -c.$$

Solving this gives $c = -1$ and

$$x = 4, \quad y = -\frac{1}{2}.$$

Substituting this into the expression for the hyperbolic paraboloid gives z

$$z = 4^2 + \frac{3}{4} = \frac{67}{4}.$$

Therefore the point is $P = (4, -1/2, 67/4)$.

2.[5pts] Suppose that the pressure in the atmosphere is given by

$$P(x, y, z) = e^{-z}(\sin 2x + \cos y),$$

and you are located at the point $(-\pi, 2\pi, 0)$ in space. In which direction $\mathbf{u}$ (a unit vector) should you move to experience the **greatest decrease in pressure**?

Solution: At any point (x, y, z) the greatest decrease in the function P is determined by the negative gradient.

$$-\nabla P(x, y, z) = -e{-}z2\cos 2x\mathbf{i} + e^{-z}\sin y\mathbf{j} + e^{-z}(\sin 2x + \cos y)\mathbf{k}.$$

at $(-\pi, 2\pi, 3)$ this becomes,

$$\nabla P(x, y, z) = -2\mathbf{i} + \mathbf{k}.$$

To make this a unit vector we find

$$\mathbf{u} = \frac{-2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{k}.$$