Math 241H - Section 0301 - Fall 2017 Name:____________________________
Quiz 6 - Solutions

You have 20 minutes to complete this quiz. No calculator, cheat sheet or aid of any kind is allowed.

1.[5pts] Set-up (but do not evaluate) the following integral

$$\iint_R \frac{1}{2\sqrt{1-x^2}}\,\mathrm{d}A.$$

Where R is the region above the x-axis and inside the circle of radius 1 centered at 0. You may use either polar or cartesian coordinates.

Solution: If we set this integral up using cartesian coordinates, the integral is actually easy to compute.

$$\iint_R \frac{1}{2\sqrt{1-x^2}}\,\mathrm{d}A = \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} \frac{1}{2\sqrt{1-x^2}}\mathrm{d}y\mathrm{d}x = 1.$$

We can also convert to polar coordinates, where the integral is not so easy

$$\begin{aligned}\iint_R \frac{1}{2\sqrt{1-x^2}}\,\mathrm{d}A &= \int_0^{\pi}\int_0^1 \frac{1}{2\sqrt{1-r^2\cos^2(\theta)}} r\mathrm{d}r\mathrm{d}\theta \\ &= \int_0^{\pi/2}\int_0^1 \frac{1}{\sin\theta}\,\mathrm{d}r\mathrm{d}\theta \\ &= \int_0^{\pi/2} \frac{1}{\sin\theta}\,\mathrm{d}\theta.\end{aligned}$$

In fact, the calculation in cartesian coordinates gives the following integral

$$\int_0^{\pi/2} \frac{1}{\sin\theta}\,\mathrm{d}\theta = 1.$$

2.[5pts] Evaluate the following integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} e^{x^2+y^2}\mathrm{d}y\mathrm{d}x,$$

by changing to polar coordinates. (Hint: first determine the region of integration).

Solution: The region is just the semi-circle of raduis 1 in the upper-half plane in polar coordinates, this becomes

$$\begin{aligned}\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} e^{x^2+y^2}\mathrm{d}y\mathrm{d}x &= \int_0^{\pi}\int_0^1 e^{r^2} r\mathrm{d}r\mathrm{d}\theta \\ &= \frac{\pi}{2}e^{r^2}\Big|_0^1 = \frac{\pi}{2}(e-1).\end{aligned}$$