Math 241H - Section 0301 - Fall 2017 Name:_____
Quiz 7

You have 15 minutes to complete this quiz. No calculator, cheat sheet or aid of any kind is allowed.

1.[5pts] Let D be the solid region inside the cylinder $r = 2\cos\theta$ between the planes $z = 1$ and $2x - y + z = 10$. Write the volume of D as an iterated triple integral in cylindrical coordinates. **Do not evaluate**. *(Hint: Be careful with the range of θ).*

Solution: The region D is described by

$$-\pi/2 \leq \theta \leq \pi/2, \quad 0 \leq r \leq 2\cos\theta, \quad 1 \leq z \leq 10 - 2r\cos\theta + r\sin\theta.$$

The fact that θ must be between $-\pi/2$ and $\pi/2$ is due to the fact that $r = 2\cos\theta$ must be non-negative. The volume in cylindrical coordinates is then given by

$$\iiint_D 1\mathrm{d}V = \int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta}\int_1^{10-2r\cos\theta+r\sin\theta} r\mathrm{d}z\mathrm{d}r\mathrm{d}\theta.$$

2.[5pts] Let D be the solid region outside of $x^2+y^2+z^2 = 1$, above $z = \sqrt{x^2+y^2}$ and below $z = 4$. Write

$$\iiint_D x\,\mathrm{d}V$$

as an iterated triple integral in spherical coordinates. **Do not evaluate**.*(Hint: A cone is given in spherical coordinates by $\phi = \phi_0$, what is ϕ_0? What is the equation for the plane $z = 4$ in spherical coordinates?)*

Solution: In spherical coordinates the cone is given by $\phi = \phi_0$, where ϕ_0 satisfies $\tan\phi_0 = r/z = 1$. Therefore $\phi_0 = \pi/4$. Also the sphere is given by $\rho = 1$ in spherical coordinates, while the plane $z = 4$ is given by

$$\rho = 4\sec\phi.$$

The region is then described by

$$0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \pi/4, \quad 0 \leq \rho \leq 4\sec\phi.$$

The integral then becomes

$$\iiint_D x\mathrm{d}V = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{4\sec\phi} \rho^3(\sin\phi)^2\cos\theta\mathrm{d}\rho\mathrm{d}\phi\mathrm{d}\theta.$$